Dynamical Systems (MATH 3410)

Lab 1 - The Computer May Lie

This lab is based on Experiment 3.6 from the textbook (p.25).

You may use Mathematica notebook files I have uploaded on the class web-page at `http://math.uni.edu/~ostapyuk/MathematicaLabs.html` or your own programs.

Consider the following three functions:

1. $F(x) = x^2 - 2$.
2. $G(x) = x^2 + c$ for some $c < -2$ (you choose c).
3. The doubling function $Doub(x)$ (you can copy-paste the formula from Doubling.nb).

1. For each of these functions, choose ten initial points (seeds). For $F(x)$, each point should be in $[-2, 2]$ and for $Doub(x)$ in $[0, 1)$. For each seed, compute the first 100 points of the orbit. Mention what happen to the orbit according to the classification in the lecture. Present your findings in the form of a table.

Note: if Overflow error occurs, assume the orbit tend to infinity; if Underflow error appears, assume the limit is 0.

2. For each of the three function, describe its overall behavior. In particular, mention what type of orbit is the most common.

3. Wait! Some orbits can be computed easily by hand. Compute exactly (by hand) the orbits of 1/3, 1/5, 1/7, 3/11 and 1/12 under the doubling function. Compare with your experimental results (you may need to experiment again if you have not selected these point in part **1.**). Explain the difference.

4. Many computer algebra systems can compute exact values of simple enough functions. In Mathematica, it can be done by removing N[ ] (which stays for "numerical") from the definition of the function and entering all values in the exact fraction form (e.g., you need to enter $\frac{43}{100}$ instead of 0.43). Redo you experiment from part **1.** for the doubling function, using exact computations. Compare with your previous results and explain the difference.

Note: do not attempt to do this for the quadratic functions, as the time needed for processing will increase significantly. You may, however, compute exact values for fewer number of iterations (e.g. for 10 instead of 100).

5. Which of the functions, quadratic or doubling, is more prone to the error build-up? Explain your reasoning.