Dynamical Systems (MATH 3410)

Lab 3 - The Transition to Chaos

This lab is based on Experiment 6.4 from the textbook (p.63).

You may use Mathematica notebook files I have uploaded on the class web-page at `http://math.uni.edu/~ostapyuk/MathematicaLabs.html` or your own programs.

The main goal of this lab is to discover the behavior of the orbit of 0 under quadratic function $Q_c(x) = x^2 + c$ for a large number of values of parameter c. In the end of the lab you will obtain a picture called **orbit diagram**, which shows an asymptotic orbit of 0 under $Q_c(x)$ (on the vertical axis) as function of c (on the horizontal axis).

1. Choose 50 different values of c as specified below. For each values of c, record the ultimate or asymptotic behavior of the orbit of 0 (i.e. the behavior after many iterates). In practice this means that we are going to skip the first 100 (or more, if you want more accurate result; but it twill take longer) iterates of zero and observe the behavior of the next 100 (or more) iterates.

Record your data in the form of the table: enter c-values in the first column and write down the ultimate behavior of the orbit (e.g. does it tend to a fixed point? To an n-cycle? Fills an interval or intervals?) in the second column.

Simultaneously, plot the asymptotic orbit on a cx-plot, as explained in part 2.

You should choose 50 c-values distributed as evenly as possible as follows:

(a) Five values in the interval $(-0.75, 0.25)$

(b) Five values in the interval $(-1.25, -0.75)$

(c) Twenty values in the interval $(-1.4, -1.25)$

(d) Five values in the interval $(-1.75, -1.4)$

(e) Ten values in the interval $(-1.78, -1.75)$

(f) Five values in the interval $(-2, -1.78)$

2. For each value of c you selected, plot an ultimate behavior of the orbit on cx-plane. E.g., if the orbit of 0 is attracted to a fixed point p, plot a point with coordinates (c, p), if it is attracted to a 2-cycles $\{q_1, q_2\}$, plot two points with coordinates (c, q_1) and (c, q_2), etc. Refer to the picture 6.16 in the textbook.

The Mathematica program provided will show you the plot of an orbit for one particular c-value. You will need to combine these graphs on one plot. Make sure your plot is large enough to show all the details (at least a page wide). Make c range from -2 to 0.25 and x range from -2 to 2.

3. After you finish the complete orbit diagram, answer the following questions:

(a) What happens at $c = -0.75$?

(b) What seems to happen at $c = -1.25$?

(c) Describe what you can see for $c < -1.25$.

(d) As you know from the lecture, a function from the quadratic family has two fixed points, one attracting and one repelling, for $-3/4 < c < 1/4$. Only one of them is seen on the orbit diagram. Which one is that? Explain why the other does not appear on the plot.

Note: you can rescale the plot in Mathematica by clicking on it and dragging it by the corner. If it is not enough, you can zoom in by changing "PlotRange" values in "ListPlot" command (currently $\{-2, 0.25\}, \{-2, 2\}$).