Dynamical Systems (MATH 3410)

Lab 6 - Exploring Mandelbrot and Julia sets

Go to "The Dynamical Systems and Technology Project" from the class web-page, then select " JAVA Applets for chaos and fractals", and then "The Quadratic Map Applet". Or just type `http://math.bu.edu/DYSYS/applets/Quadr.html` in your browser. Note that Java may not work due to security settings, then you will need to add `http://math.bu.edu` to the exception site list.

You will see the Mandelbrot set on the right and the filled Julia set for the selected value of parameter c on the left. You may enter the new value of parameter c in the window in the center or just click in the Mandelbrot set picture.

1. By definition, the Mandelbrot set is the set of values of the parameter c such that the orbit of 0 does not go to infinity under $Q_c(z) = z^2 + c$ map. There is an equivalent definition of the Mandelbrot set, based on the connectedness of the filled Julia set. Select 5 values of c in the Mandelbrot set and 5 values of c outside it. Observe the properties of the corresponding filled Julia set. Is is connected, disconnected, totally disconnected? (Recall that the set is connected if for any two points in the set there is a curve in the set connecting them, roughly speaking, if the set contains one piece only. Otherwise, the set is disconnected. The set is totally disconnected if no two points can be connected by a curve within the set.) Present your findings in the form of a table. Make a conclusion about the Mandelbrot set and the connectedness of the filled Julia sets.

2. What happens if we move away from the Mandelbrot set? Select 5 values of c outside the Mandelbrot set, with the first value being very close to the boundary, the second is farther away and so on. Observe what happens to the filled Julia sets. Write down your conclusion.

3. The symmetry: you should observe that the Mandelbrot set is symmetric with respect to the real line (x-axis). Try to find out what will happen for the filled Julia sets for the symmetric values of parameter c. In particular, select 10 values of parameter c, both inside and outside of the Mandelbrot set. Make sure to select various values in different places. Then consider the filled Julia sets for the value c and its complex conjugate $\bar{c}$ (recall that if $c = x + iy$, then $\bar{c} = x - iy$). Do not click on the picture, but rather use the manual entry of c-values to insure accuracy. What do you observe? Write down your conclusion.

4. Explore the properties of the various "pieces" of the Mandelbrot set. You can observe that the Mandelbrot set consists of several (actually, not several, but infinitely many!) "pieces", the largest one being an interior of a cardioid, surrounded by the smaller ones, usually called "bulbs" or "decorations". The filled Julia sets corresponding to every "piece" have very specific properties. Try to discover these properties by selecting at least 5 different c-values form each of the following "pieces":

(a) the main cardioid;

(b) the largest bulb directly to the left from the main cardioid;

(c) the next largest bulb approximately on the top of the main cardioid;

(d) the next largest bulb between the bulbs in parts (b) and (c).

What is the common feature of the filled Julia sets for each of these "pieces"?