Complex Analysis I (MATH 6460)

Final Exam - theoretical part, Fall 2017

Make sure every statement is explained. You may cite and use all properties and theorems from the part of the textbook we have covered and homework exercises.

1. Prove the following identities for any z in the domain of the functions:

(a) $\mathrm{Log}(\bar{z}) = \overline{\mathrm{Log}(z)}$

(b) $\sinh(\bar{z}) = \overline{\sinh(z)}$

(c) $\sin(\bar{z}) = \overline{\sin(z)}$

2. Let f be an analytic function on $\mathbb{C}$. Show that the function $g(z) := \overline{f(\bar{z})}$ is also analytic.

3. Let u be a (real-valued) harmonic function, v be the harmonic conjugate of u and $\tilde{u}$ be the harmonic conjugate of v. How are u and $\tilde{u}$ related? Prove your claim.

4. Let f and g be entire functions that satisfy

$$|f(z)| \leq |g(z)|,$$

and g has no zeroes.
Prove that there exists a complex number C such that $f(z) = Cg(z)$.