

## Additional HW problems for Chapter 3 for graduate students.

Required for graduate students, other students may attempt it but no credit will be given. Turn in separately from the textbook assignment.

**1. (a)** Prove that if  $\{x_n\}$  is an orbit of continuous function  $f$  and  $\lim_{n \rightarrow \infty} x_n = a$ , then  $a$  is a fixed point of  $f$ .

**(b)** Provide a counterexample to the statement (a) when  $f$  is not continuous.

**(c)** If  $\{x_n\}$  is an orbit of continuous function  $f$ ,  $\lim_{n \rightarrow \infty} x_{2n} = b$  and  $\lim_{n \rightarrow \infty} x_{2n+1} = c$ , what can you say about points  $b$  and  $c$ ? Prove your statement.

**2.** Show that an orbit of any rational point under the doubling function is eventually periodic or eventually fixed.